Segregation and gentrification as emerging phenomena from the real estate market

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Abstract The complex character of the gentrification is recognized, in which both the economic aspects, as they emerge from the demand and supply side explanations, and the segregation dynamics are included. After having analyzed the modeling approach to the gentrification, a model based on the micro-economic theory of the residential location is proposed. The model's dynamics occurs in a classic mono-centric city. Two classes of agents are responsible for this dynamic. These are: developers which build and rehabilitate, and households which bid for buildings with a variable income, and a variable frequency of movement. Buildings are carried out by developers and sold in an auction market. Household's bid depends on income, and on transportation costs, as in the classic Alonso-Muth model relying on von Thünen. Households with low income may gather in order to sum up incomes and bids, so that a variability of crowding of dwellings can occur as well . Because the building's quality decreases naturally, degraded areas emerge in connection with poor households. The rehabilitation is performed either by households (considered as owner-occupiers) or by developers (including landlords). The gentrification of the inner areas is observed in relation to the main parameters of the model. Conclusion are drawn on the complexity of the gentrification, and how the parameters (owner-occupier and tolerant households percentage, and the preference for low density housing) influence this phenomenon.

Key words: real estate market, urban segregation, gentrification, urban dynamics, urban modeling

D44 - Auctions, R14 - Land Use Patterns, R21 - Housing Demand, R31 - Housing Supply and Markets, E17 - Forecasting and Simulation

1 Introduction

Gentrification is the displacement or replacement of a low ranking socio-economic group by a higher status socio-economic group in the inner city, and it involves the renovation of previously downgraded buildings for residential use. The theories about why gentrification occurs are usually divided into demand side theories and supply side theories. Demand side theories focus on changing preferences [8] that might led to an increased demand for high income groups for centrally located housing. In turn, the supply side theories are related to the "gap theories" focusing either on a rent gap [15],[4] or on a value gap. A gap exists in an urban area, when the current rent of property value is significantly inferior to the potential value of the property.

This gap may depend on the abandonment of the inner city and the later inflation of the cost of suburban housing. This situation creates an opportunity for developers to make profits by renovating inner-city buildings for residential use [9].

Even if sometime disregarded in relation to gentrification, the segregation dynamics does play an important role. Segregation is the kind of effect that takes place in residential areas in large cities, where people wish to be surrounded by kindred neighbors. The dynamics of segregation has been studied as an emerging phenomenon related to certain cultural or ethnic dichotomies of the population [12]. The phenomenon of gentrification, which is not always related to the ethnic origin of the inhabitants [10], can sometimes be explained by class [2], and may be the result of a particular kind of residential segregation. In fact, in the analysis of the gentrification phenomena it is found that people identify with particular neighborhoods, look for those where "people like us" would live [3], and where they can participate in the cultural milieu and take advantages of job or education opportunities [18]. Segregation appears thus both as the result and as a reinforcing factor of the gentrification.

However, to look for one specific cause of the phenomenon may be misleading because, as recognized, urban dynamics is a complex phenomenon emerging from a myriad of actors involved. This is what the modeling approach, presented in the next section, is pinpointing.

2 The modeling approach

Although the modeling approach represents a very limited part of the huge literature about gentrification, it stresses an important aspect, namely that of the complexity of the phenomenon.

The first basic model is that of Wu, [17] who has simulated the development of a cellular automata model, which may represent the gentrification process viewed from the supply side. Each cell hosts a variable amount of population. The addition of a unit of population happens when the difference between the average density of the neighborhood and of the central cell reaches an established threshold: a situation called "niche". Since the addition of a unit of population may give way to further niches, the possibility is open for large avalanches of new population, when the system reaches a critical state, such as in the SOC [1]. These niches represent for investors an opportunity for potential profits. The investment, i.e. the addition of an unit of population, may generate new niches, so that the system is in a continuous state of instability.

While in the Wu's model the population is continually growing, a more realistic model of gentrification has been proposed by Diappi and Bolchi [5] [6]. This model is based on cellular automata as the previous one. The key variable in each cell is the rent of the dwelling built in it. The rent decreases during the simulation due to the natural degradation of the building, so that a gap may result between the rent in a cell and that of the surrounding ones The approach is similar to that of Wu: investments happens where a difference greater than an established threshold between the rent in the central cell and the average rent in the neighborhood does exist. While in the previous model the fluctuations of investments result from the dynamics, in this model the amount of investments is a parameter, so that the model may simulate periods of high or low availability of capital for real estate investments.

In contrast with the previous models which are, in essence, supply oriented, the models of O'Sullivan and Torrens present a demand-supply dynamics. In addition instead of considering the whole urban structure these models focus in one part of the inner-city. People get in and out from the neighborhoods, so that the gentrification occurs when the socio-economic status of

the entering people is higher that that of the exiting one and dwellings are rehabilitated. In the O'Sullivan model [11] a cellular automata dynamics is utilized and each cell is characterized by the inhabitants income, as well as by the properties quality. Households may decide to leave the area applying a segregation criterion: households who find themselves in relatively inferior properties, or whose income is well above that of their neighbors, are more likely to move out of the area. Incoming households behave according to a similar method: if the property value is locally high then the incoming household's income will be the same as the wealthiest neighbors. Households whose property is in poor condition relative to their neighbors may upgrade their house in dependence of the loan availability which are preferentially provided to higher income residents.

The model of Torrens and Nara [16] shows a similar dynamics embedded in a multi-agent simulation. The structure of the model is more complex in that involves agents, and real estate market. In their model, each resident, having heterogeneous attributes, determines the mobility of choosing a property or leaving his/her property by a probability function, which consists of individual's preferences and introduces a hierarchical nested choice structure. As a function for properties, property values are updated by examining the vacancy rate of a property and its neighborhood, which is dynamically altered by the residential mobility.

These models, in their effort to reproduce the gentrification in a more or less synthetic way, emphasize its complex character. Namely that it emerges from the economic interaction of a lot of actors. Because of the economic character of the gentrification, we want to propose a model which is based on the micro-economic theory of residential location. In fact gentrification is nothing but a phenomenon of renewal and/or rehabilitation of the real estate, in connection with a upgrading of the socio-economic status of the occupiers, so that the two aspects–status of property and of occupier– reinforce themselves. Households that cannot compete with the bid of higher economic class households are compelled to accept a low standing flat eventually in another part of the city, where their bid is the highest. In essence, we want to show how the application of the basic micro-economic theory, as emerging from the work of Alonso and Muth is able to generate the phenomena included in the literature on gentrification [7].

3 The model

We present a multi-agent model based on the Ricardian rent theory, as developed in a spatial agricultural context by von Thünen, and later extended to the real estate urban market by Alonso-Muth theory of residential location. The model is intended as a part of a more complex model, CityDev [14], based on the framework: agents-goods-market. The model is a simplified version in order to better understand the basic dynamics. Here in fact, we consider a monocentric city in whose center are located the commercial facilities and the workplaces.

In the model, which is spatially organized in a grid, two classes of agents are included: developers (large corporate interests down to locally operating landlords) which carry out and rehabilitate buildings, and households which bid for properties in the real-estate market and eventually directly rehabilitate them. At each step the number of agents increases and consequently the number of buildings increases as well. The growth is logistic, but, after the saturation is attained, in order to test the functioning of the model independently from the growth of the system, a long period of stability is simulated.

In essence the dynamics is sketched as follows. Dwellings are carried out and supplied by developers, households bid for dwellings in the real estate market which are assigned to the

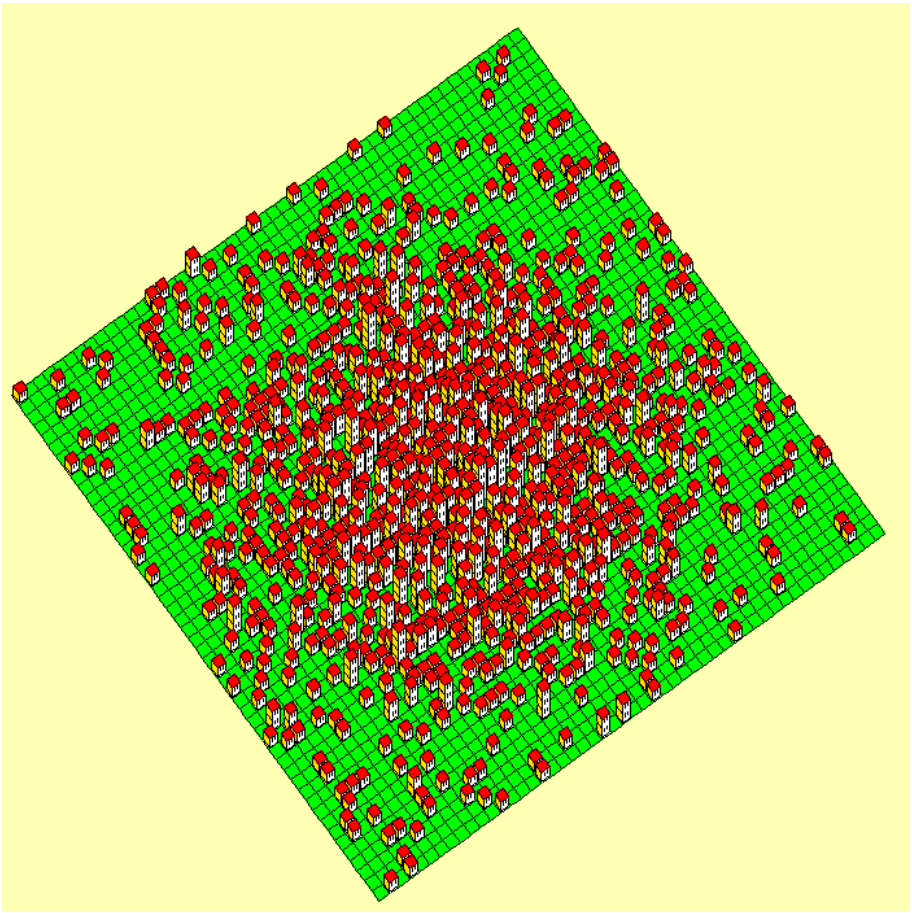

Figure 1: The urban fabric in which households are located in an axonometric view. Activities are supposed to be located in the city center, even if any space is reserved for them.

best bidders. Dwelling naturally degrades, and a household may abandon it. When abandoned a dwelling may be rehabilitated either by the household-occupier after the assignment, or by a developer before it is put up for sale in the market. An in-depth analysis of this process is performed in the next paragraphs.

Properties are sketched as cubic cells that can be over-posed on a squared ground cell. In the model these cubic cells are built by a developer according to a maximization of an utility which increases with the accessibility to the city center and decreases with the local density (see [13], and figure 1 for an example of the resulting urban fabric). Each property has a variable floorspace, which randomly increases with the increase of the distance from the city center, and a quality which naturally decreases during the time, so that a degradation of the building occurs.

Households represent people living and working in the city, which need a dwelling. The main attributes of a household are: the income Y_i which, for sake of simplicity, is totally devoted to the housing rent and transport costs, the frequency f_i a household goes to shop, the weight θ_i representing the preference for a low density housing, and the tolerance or the intolerance in relation to the difference in income with the surrounding households.

Heterogeneity characterizes the households. In fact the income is distributed according to a power law, while the frequency is distributed at random with a maximum and a minimum, inde-

pendently from income. In turn the degree of preference for low density housing is connected to the frequency of movement. It is supposed that a low frequency is associated with a high preference for low density, and an high frequency with a low preference for low density. Tolerance and intolerance are distributed at random according with an established probability p_θ which is a parameter of the simulation. In addition a probability p_ρ is set in relation to the rehabilitation of the dwelling: with this probability a household may behave as an owner-occupier and rehabilitate directly the house. Note that the probability a developer considers an abandoned property for rehabilitation is $1 - p_\rho$, so that p_ρ represents roughly the share owner-occupiers households.

These attributes, which locate the household in the social space, affect the behavior of the household. In addition, they are utilized to calculate the social distance between each couple of households, which is utilized for two goals. First, to couple two housholds looking for a dwelling in case their income is is not able to guaranty a dwelling. Households are coupled beginning from that with the smallest social distance. One of the two is considered as the chief which bids in behalf of the other, summing up the two budgets. The above mentioned method helps in simulating the common strategy diminishing the requested quantity of floorspace, so that the bid per unit of floorspace increases. In fact the two coupled households share the flat, hence lowering the floorspace per each of them. Second, the social distance is utilized to establish a network of friends (maximum ten) whose situation is compared with that of the household in case of decision to abandon or not the dwelling.

In order to get the wished dwelling, households bid in the real estate market. Because the rehabilitation by a household happens immediately after the dwelling has been assigned, the choice is twofold: to bid for the property as it is, or to bid with the intention to rehabilitate it, after it is assigned to the household. The household makes the first choiche in case the property has a quality over an established threshold. According to the theory of residential location, the bid for a dwelling is the income minus transport cost which are divided in shopping and commuting. In conclusion a bid of a family is given by:

$$B_{ij} = Y_i - c(f_i S_{ij} + C_{ij}), \tag{1}$$

where: B_i is the bid of family i for the location j, Y_i is the income of family i, c is the transportation cost per unit of distance, f_i is the frequency of shopping, i.e. the times the household i goes to shop in a step, S_{ij} is the traveled distance to go to shopping in relation to the location j, and C_{ij} is the traveled distance for commuting in relation to the location j. The second choice is made in dependence with p_ρ the probability a household behaves as owner occupier instead of as a renter. In addition the quality of the property should be under an established threshold, the bid should result higher than the current value of the property as calculated on the base of the rent, floorspace and quality of the surrounding properties. In this case the bid is the following:

$$B_{ij} = Y_i - [c(f_i S_{ij} + C_{ij}) + r(F_j)], \tag{2}$$

where r is the rehabilitation cost per unit of floorspace which is considered as constant.

In order to consider a process of bargain, in case the bid of the household is higher than the rent payed by the the previous occupier, the bid id recalculated as follows:

$$B^\star_{ij} = \frac{B_{ij} + R_j}{2}, \tag{3}$$

where $B^\star_{ij}$ is the final bid of the household i for the property j, and R_j is the rent paid by the previous occupier. Further the household evaluates the property from its point of view, and

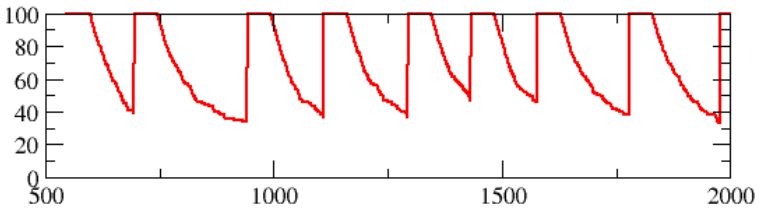

Figure 2: The variation of the quality of a cell taken at random during the simulation. X axis: steps of the simulation. Y axis: quality.

considers the theoretic gain realized for each supplied property as the difference between the value of the property and the bid the household is willing to pay for it. Later the household will bid for the property resulting in the maximum gain.

The evaluation takes in account the floorspace and quality of the building, the income of the surrounding households, the density of the neighborhood, and the accessibility to shops and working place. This evaluation is calculated in the following way:

$$E_{ij} = F_j Q_j D_j^{\delta_i} T_{ij} \exp[-\beta(f_i S_{ij} + C_{ij})], \tag{4}$$

where E_{ij} is the evaluation of the property j by the household i, F_j is the floorspace of the property, Q_j is the quality of the property, D_j is the density of the neighborhood, δ_i is the weight the household assigns to the previous factor, T_{ij} is a factor connected with the difference between the income of the household and that of the surrounding ones, and $\exp[-\beta(f_i S_{ij}+C_{ij})]$ is a measure of the accessibility of shops and workplaces from j.

Let us consider the different factors included in the calculation of the evaluation. Each dwelling, once carried out, has a quality which is set to the maximum. Later the quality Q_j decreases during the steps of the simulation, so that it is dependent on time. In fact, after a period in which the quality is stable, it decreases with an established probability at each step. In case of rehabilitation (see later) the quality is set newly to the maximum (see figure 2, where a sample building's quality during the simulation, is represented). The density is calculated in the following way:

$$D_j = 1 - \frac{N_j}{\max_j(N_j) + 1}, \tag{5}$$

where N_j is the number of the surrounding buildings in a 3×3 neighborhood, and $\max_j(N_j)$ is the maximum of the previous measure to which 1 is summed up in order to prevent D_j takes a zero value. Thereby D_j increases with the decrease of the density, reaching the maximum in case of an isolated building. The factor T_{ij} represents the tolerance (or the intolerance) of the household i. In fact in case the household i is tolerant, $T_{ij} = 1$. In case of intolerance, the factor is calculated in an asymmetrical way: if the income Y_i is lower than the average income $\overline{Y_j}$ of the neighbors surrounding the property j, then $T_{ij} = 1$, otherwise its is calculated as follows:

$$T_{ij} = \exp[-\theta(Y_i - \overline{Y_j})], \tag{6}$$

where θ is a parameter which represents the degree of intolerance: the greater its value, the greater the intolerance in relation to the poorer housholds. In simple words, while the poor households have any difficulty to be located near the rich ones, the rich households do not wish poor households around them. In so doing, the segregation dynamics is totally included in the economic functioning.

After having evaluated the property, a household is ready to calculate the gain G_{ij}, such as follows:

$$G_{ij} = \gamma E_{ij} - B^{\star}_{ij}, \tag{7}$$

where γ is the average ratio: rent payed by housholds, evaluation of the property, so that G_{ij} is expressed in monetary value. Note that in case of intention to rehabilitate this gain includes the expenses for rehabilitation:

$$G_{ij} = \gamma E_{ij} - [B^{\star}_{ij} + r(F_j)]. \tag{8}$$

In conclusion the household bids for the property connected with the maximum G_{ij}, according with the rules of the real estate market.

The real estate market is organized as an auction market, in which the supplied properties are assigned with a recursive method. In fact, at each turn, a list of the supplied properties in the market is proposed, and each household calculate the bid, the evaluation and the gain and chooses the property to bid for. The bids for each property are collected, the property is assigned to the highest bidder, and the rent is set equal to the bid:

$$R_j = \max_i(B^{\star}_{ij}). \tag{9}$$

In case the winning household has planned to rehabilitate the property, the quality is set to initial quality and the rent of the building is set to the bid plus the cost of the rehabilitation. The property in question is canceled from the list of supplied buildings and a new turn of the market newly begins. In the next turn the method is repeated, for the properties not yet bought and for the households yet looking for a dwelling. The process stops when in the previous turn any property has been assigned.

Dwellings are abandoned for two reasons. First because the probability to abandon increases with the period the agent stays in the dwelling. Second because the dwelling is not adequate for the agent, for instance because the average income of the surrounding residents has changed. In the abandon of a building it is crucial the knowledge obtained trough social relationship. Each agent is supposed to be in relation with ten friends which are chosen with a criterion of similarity in relation to the income and frequency of movements. The agent compares its situation (price and quality of the housing) with the average situation of its friends. In case of high difference, the probability to leave increases.

The abandoned properties are considered by developers for rehabilitation. The decision of a developer to rehabilitate an abandoned dwelling depends on its quality which should be lower than an established threshold, by the difference between the average quality of the surrounding dwellings and that of the dwelling in question, which should be higher than an established threshold. In addition the developer evaluates the difference between the foreseen new rent of the property in case of rehabilitation, as calculated on the base of the rent, floorspace and quality of the surrounding properties, and the rent that the last occupier was paying. The rehabilitation by a developer happens when this difference is higher than the cost of rehabilitation, and with a probability $1 - p_\rho$. Remembering that p_ρ is the probability a household behaves as owner-occupier, the prevalence of one of the two types of rehabilitation depends on the percentage of rentals and owner-occupiers which, of course, is an important parameter of the model.

4 Results

The model has been experimented in a theoretic situation of a city with 1500 households, located on a grid of 50×50 cells. The simulation runs for 2000 steps. The first 1000 steps are utilized for

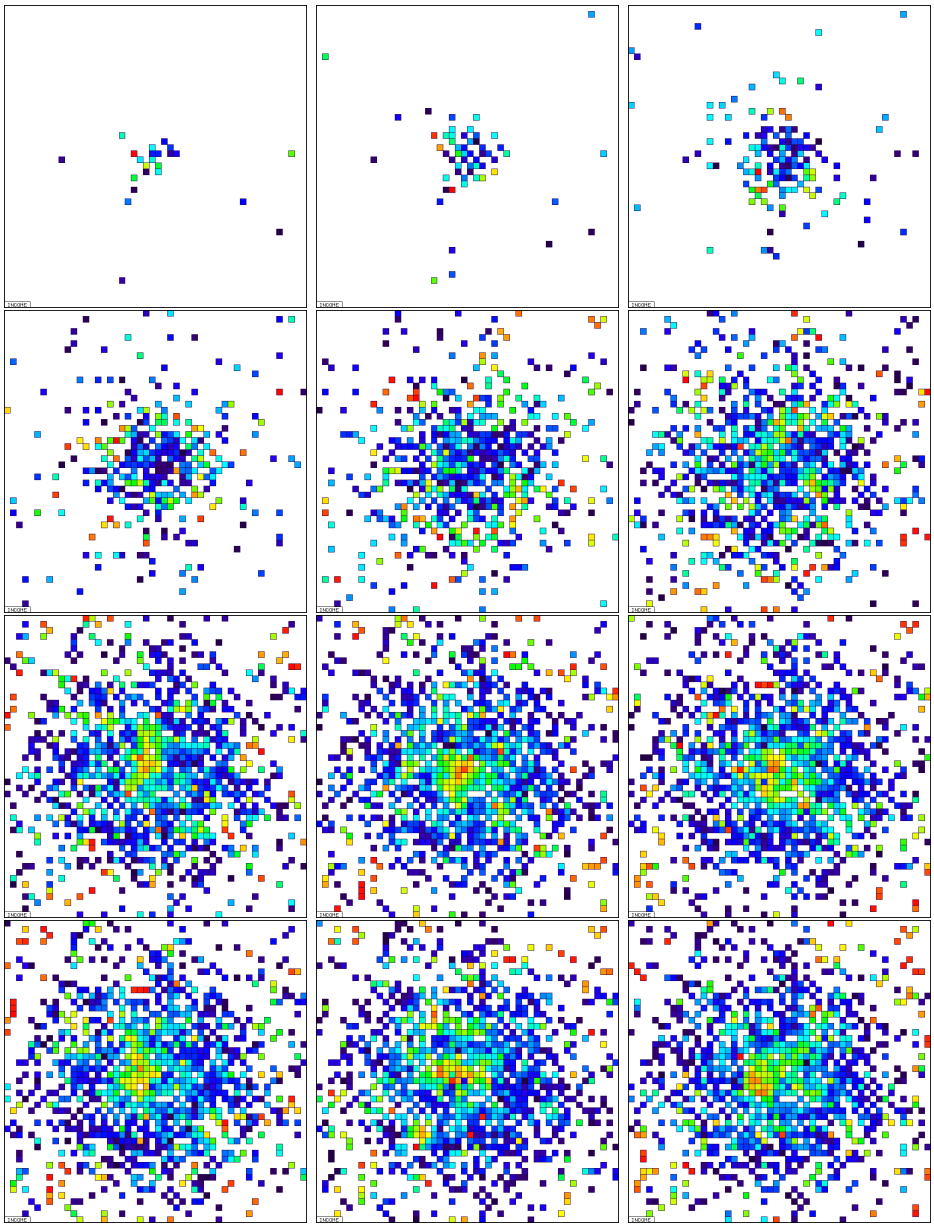

Figure 3: The spatial distribution of inhabitants by income during he simulation. (From top-left to bottom-right, the steps represented are: 100, 200, 300, 400, 500, 600, 750, 950, 1200, 1450, 1700, and 2000). High income households: red colored, low income: blue colored.

the logistic growth from 3 household to 1500. Hence, after this first period of growth, a second period of stability in population is simulated. The urban fabric obtained with the simulation consists of cubic cells over-posed on squared cell (figure 1), so that the density decreases from the center of the city till the suburb.

To understand the functioning of the simulation the straightforward approach is the analysis of the resulting spatial distribution of household by income. In fact from the distribution of income one understands the level of segregation and the phenomenon of gentrification by

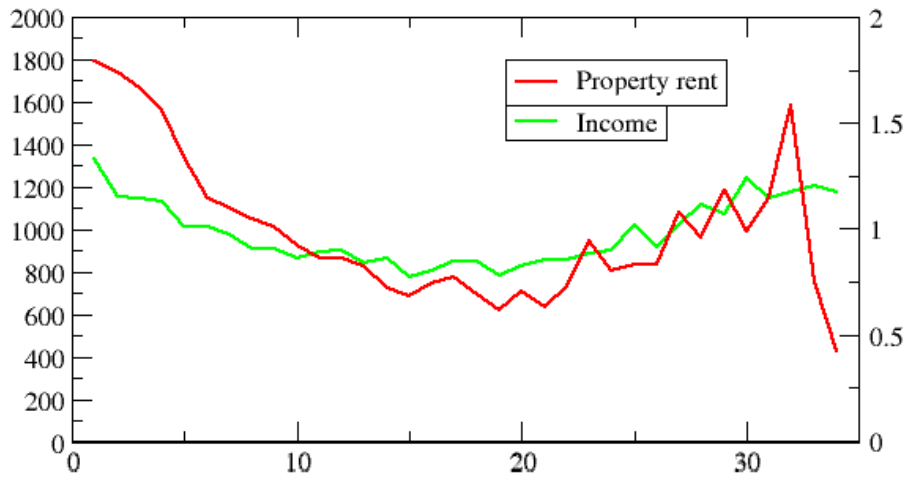

Figure 4: Average income and average property rent per unit of floorspace, as a function of the distance from the city center. X axis: distance from the city center. Y axis: Left, average income, right, average property rent.

comparing two distributions related to sequential steps. As figure 3 shows, after a first phase in which the high income households are compelled to locate in the suburb (figure 3, first and second rows), due to the decrease of the quality of the inner-located properties, which were built in the first period of the simulation, it follows a second phase in which the inner city is invaded and gentrified by high income household (figure 3, third row). However, households having a low frequency of movement and a high preference for low density areas, are located in the suburb. In the third phase which lasts for a long part of the simulation (figure 3, fourth row), the high income households continue to gentrify the inner areas.

The final distribution of the average income and of the average property rent as a function of the distance from the city center, can be appreciated in figure 4. The high income households gather near the city center, and are diffused in the peripheral areas. This is due to the distribution of the frequency f_i, so that households with and high frequency are located in the inner city, while that with a low frequency in the suburb. Thereby the property rent first decreases with the increasing of the distance from the city center and later increases. Between the center and the suburb a circular zone of medium income and medium rent is located.

In order to highlight the role of the three most crucial parameters of the model, six experiment have been performed assigning extreme values to the following parameters: the probability p_ρ that a household behaves as an owner-occupier, thus rehabilitating the propriety immediately after it has been assigned, the probability p_θ an household is tolerant, and the weight δ_i representing the level of preference for the low density housing. All the other parameters remaining equal, to the above mentioned parameters the following values have been assigned. In the first couple of experiments the p_ρ parameter takes the value 0 (rehabilitation carried our by developers), and 1 (rehabilitation carried out by households); in the second couple of experiments the p_θ parameter takes the value 0 (all the households intolerant) and 1 (all the households tolerant); and in the third couple of experiment the δ parameter ranges first $0 - 0.1$ (low preference for low density housing) and second $0.2 - 0.4$ (high preference for low density).

From the different spatial distributions of households by income at the end of the simulation (figure 5) some conclusion can be drawn.

When the rehabilitation is carried out by developers the inner city location of the high in-

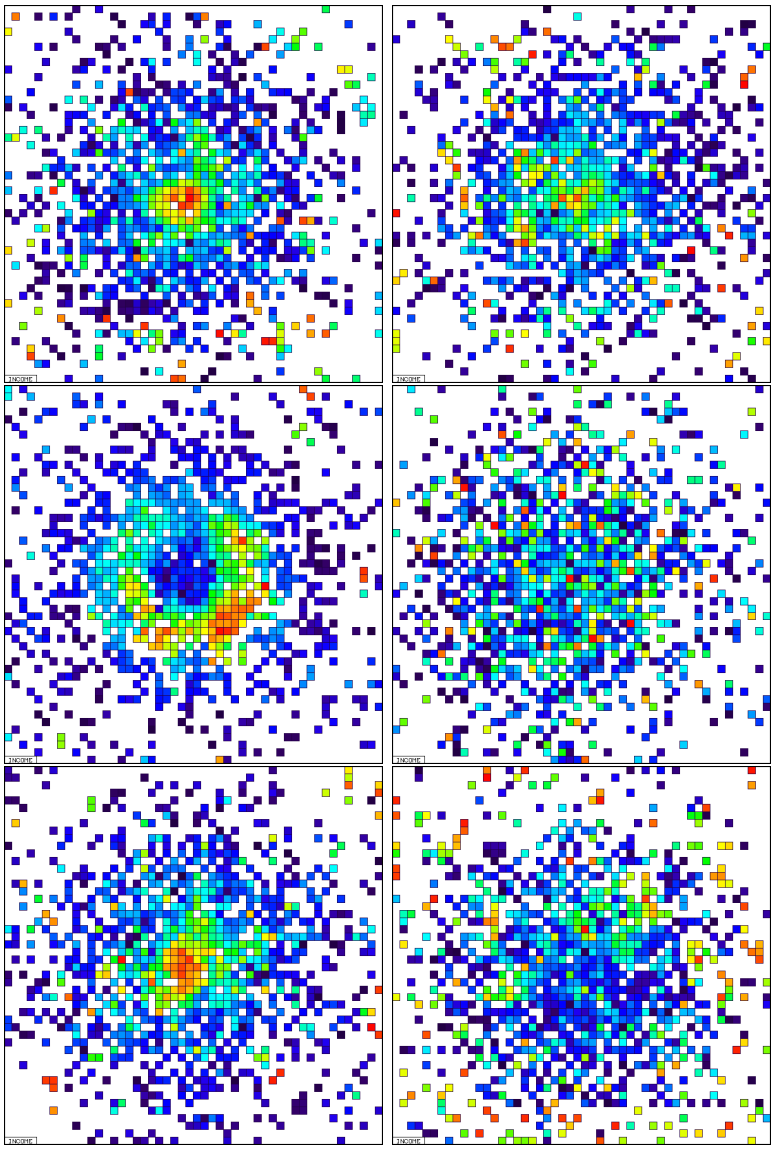

Figure 5: The spatial distribution of inhabitants by income at the end of the simulation. High income households: red colored, low income: blue colored. First row: variation of the probability an household behaves as an owner-occupier. Left: the rehabilitation is carried out by developers, right by households. Second row: variation of the share of tolerant households. Left: all the households are intolerant, right: all are tolerant. Third row: variation of the weight assigned to the low density. Left: low preference for low density, right: high preference for low density.

come household is emphasized (figure 5, first row), even if the index of gentrification during the simulation is the lowest (figure 9), meaning that the investments of developers are limited by the number of situations without risks, so that the rehabilitation by owner-occupier may result in a triggering function. This aspect is emphasized in the figure 8 which shows how the average quality is lower in case of rehabilitation performed by developers in comparison to that performed by households.

The role of the tolerant households is highlighted by the second couple of experiments.

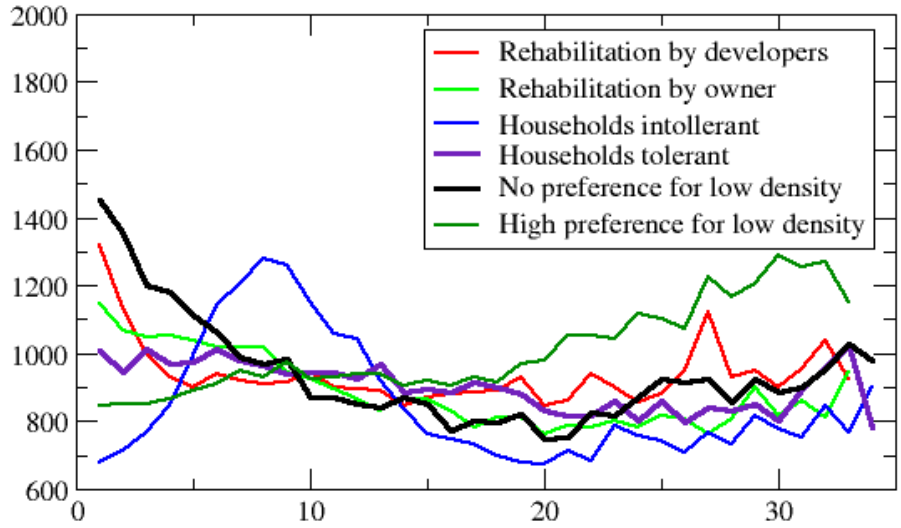

Figure 6: Average income as a function of the distance from the city center. X axis: distance from the city center. Y axis: average income. Comparison among the six experiments.

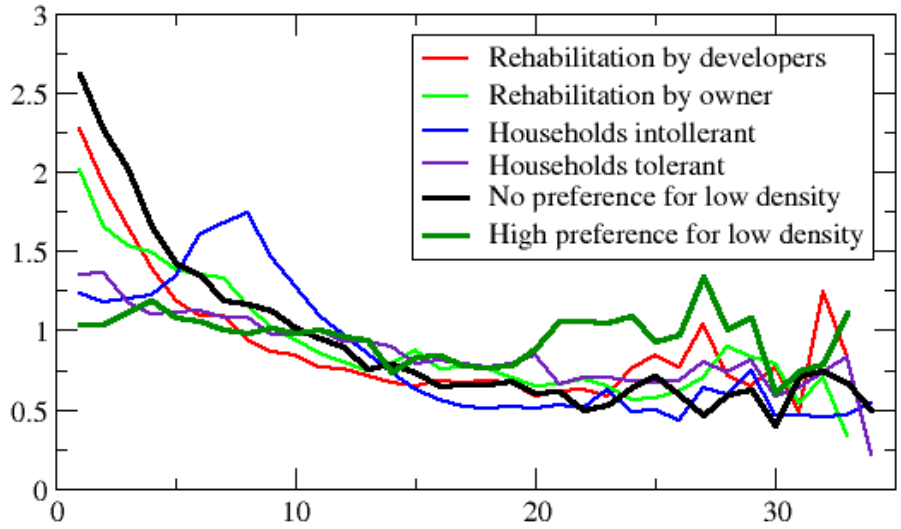

Figure 7: Average property rent per unit of floorspace, as a function of the distance from the city center. X axis: distance from the city center. Y axis: average property rent. Comparison among the six experiments.

In case all the households are intolerant the inner city is a degraded area inhabited by poor household (figure 5, second row), while, in case of tolerance, the phenomena of gentrification are amplified (figure 9), with an important increase in the average quality (figure 8) even if without any spatial concentration. The distribution of property rent shows in the first case the higher values in the intermediate zone, while, in the second case, the rent decreases slowly (figure 7).

The results of the variation of the preference for low density housing are shown in figure 5, third row. In case of low preference for low density housing, high income households are

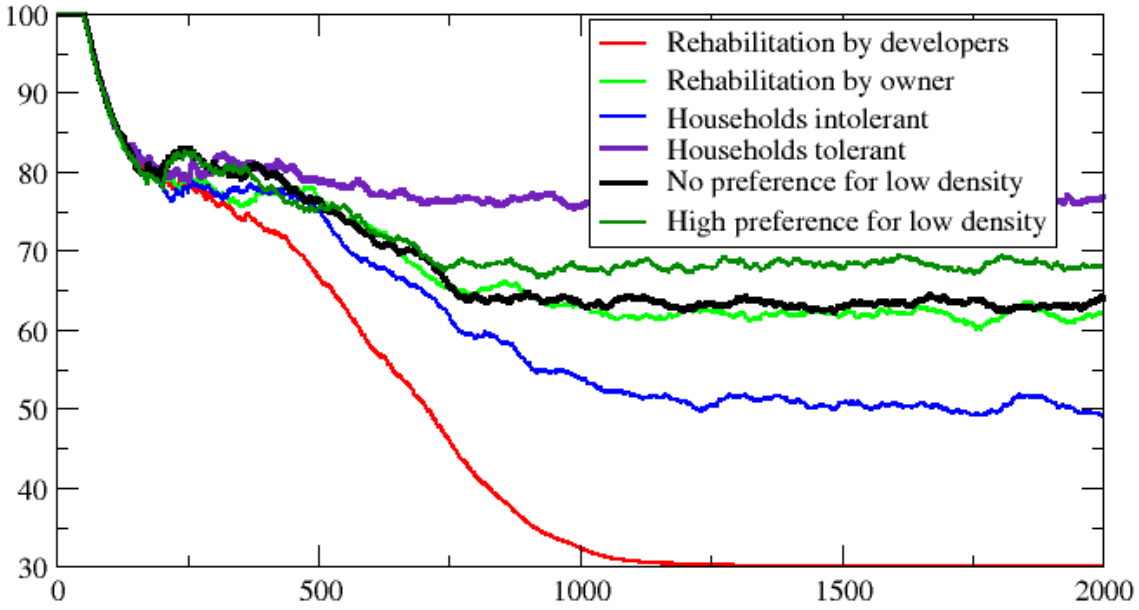

Figure 8: The average quality of properties during the simulation. X axis: steps of the simulation. Y axis: quality. Comparison among the six experiments.

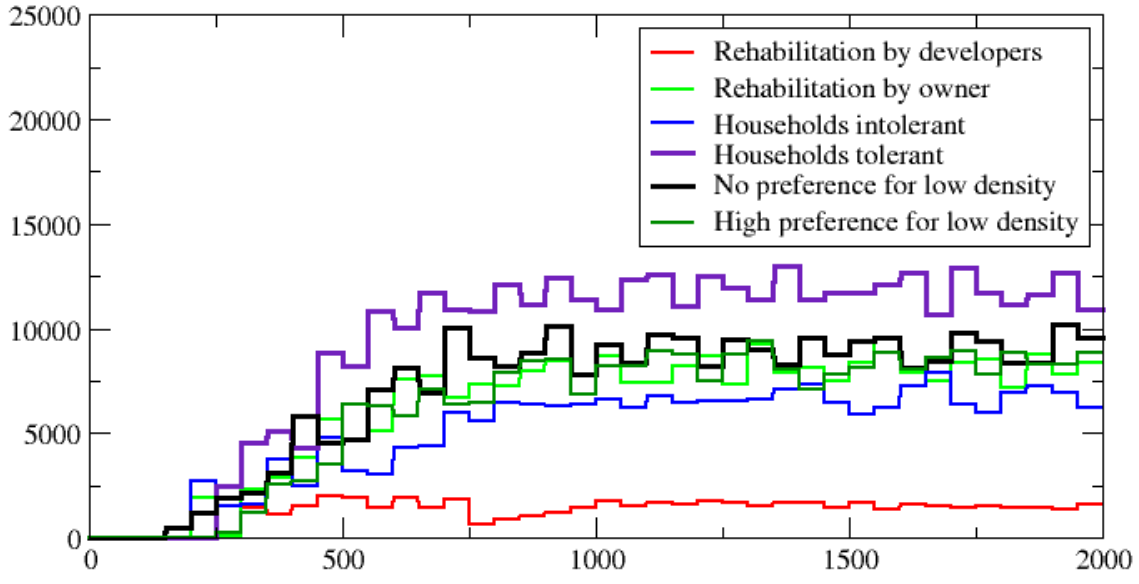

Figure 9: The index of gentrification during the simulation. X axis: steps of the simulation. Y axis: index of gentrification. Comparison among the six experiments.

gathered in the inner city, while in case of high preference for low density the high income housholds are mainly located in the suburbs. The distribution of the property rent highlights the high rent vale in the center, in contrast with the flat distribution in the second case (figure 7).

5 Conclusions

We have shown that a gentrification is a complex phenomenon that can be simulated using the micro-economic theory of residential location, in connection with the segregation dynamics. This theory, applied in a multi-agent simulation, is able to reproduce the classic decaying of the rent as a function of the distance from the city center. The gentrification results from the interactions among the dynamics of rehabilitation, the competition for land use and the willingness of rich people to be surrounded by similar inhabitants. In addition, the following aspects are crucial parameter of the model: the share of owner-occupiers, the percentage of tolerant household, and the preference for low density. The owner-occupiers are an important component for the activity of rehabilitation even if developers are responsible for the concentration of the rehabilitation in the inner city due to their strategy which avoids major risks. The tolerant household are the catalyst of a gentrification process, which is strictly connected with a low preference for low density housing. In conclusion to analyze the gentrification as a complex system is emerging as a new approach, able to trow light also in the control of the phenomenon for planning purposes.

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