Self-organized criticality in urban spatial development

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Abstract

The micro-dynamic models of urban development, usually conceive the evolution as a continuous process of diffusion. Nevertheless, in many cases the changes of the urban fabric depend on a chain of causation which gives rise to a great number of little projects and to a very few number of great urban projects. In this paper I present a model simulating the urban development which highlights these phenomena. In fact, in this model the dynamic depends on the accumulation of a potential energy which is suddenly released. In addition, a reaction chain is stimulated by a diffusion process in the neighborhood such in the sandpile model. The model is developed in a 3-D spatial patter, composed of cubic cells which take a limited number of states: un-built, housing, retail and industry. The changing of state happens when the potential energy accumulated overcomes an established threshold, and depends on local and global causes. The global causes are responsible for the accumulation of energy. In turn local causes stimulate the reactions chain resulting in the urban avalanche. The model is experimented both in a growth period, and in a stagnation period. The power law distribution of urban avalanches is analyzed. A parameter is further applied to the effects of the chains of causation, and the results obtained with the variation of the parameter are evaluated in relation to the the sensitivity to the initial conditions.

Introduction

The growth of an urban cluster is usually conceived as an addition of some elements to the existent cluster in relation to the state of the elements in the spatial neighborhood in the previous step. Nevertheless in many cases changes happen simply by imitation of previous changes. In other words, the elements change their state in relation to the state of the surrounding elements, as well as in relation to the variation of it. This functional relation generates the domino effect: the falling down of one element is able to originate a chain of variations which can continue ad infinitum (figure 1). In the urban dynamic these phenomena may happen in a planned or unplanned way: a gentrification process is a typical unplanned transformation of an entire urban area. In other cases huge transformations are planned and the building or the renovation of an urban area can be completely designed, even if it is usually supposed a start up of the project, for instance the investment of a public company, and a following process of imitation by other investors. The dynamic of a city is characterized by these chains of imitation which give rise to a set of transformation involving urban areas of different size, and the size distribution of these urban areas is similar to a power law distribution. In other words the stable state of a city is a critical state in which projects without a typical size can be produced [5]. Even if Alexander [1] has anticipated such vision of the urban dynamic at least for designing purposes, the theoretic background of the present approach refers to the theory of self-organized criticality which

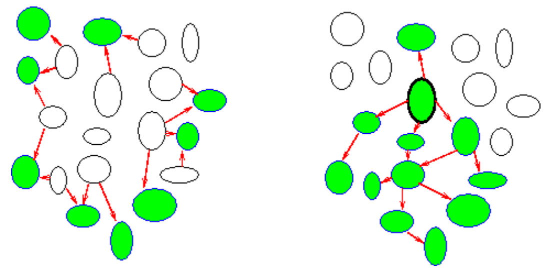

Figure 1: The addition of cells versus the chain of changes.

was formulated by Bak et coauthors ([2]). The sandpile model, utilized in order to study the properties of similar systems, is resumed hereafter. This model is usually experimented in a 2-D space, organized in squared cells which can take two states, say 0-1. In this model a cell at random receives a grain. This action is considered as a perturbation of the system. When the number of grains in a cell overcomes an established threshold, the cell changes state and the grains located in the cell are distributed in the surrounding four cells. Normally the threshold is equal to the number of cells in which grains are redistributed. By using this method in each surrounding cell only one grain is received from the cell which has changed its state. The falling down of the grains is suspended if the number of grains in one cell attains the threshold. In this case, in fact, a chain of changes, generally called "avalanche", begins. In other words the number of perturbations is reduced to the minimum and each avalanche is not connected with the following. In fact after a site has changed state it comes back to the previous state and is ready to be eventually invested by the following avalanche.

From the point of view of the urban dynamic, the grains can be conceived as the opportunities to invest due to the increase of land rent. When these opportunities overcome a threshold the cell is built, thus influencing the surrounding cells. The potentiality to invest is zero in the central cell after the investment was performed while the opportunities of the surrounding cells increase because the risks decrease. Anyway it is not possible to completely transfer the sandpile model in the explanation of the urban dynamic. In fact in the urban dynamic a site is frozen if it is invested by a building process, at lest for an established period which, in essence, depends on the time needed for the amortization of the investment. For this reason not all the areas of a city are in a critical state. In addition in the sandpile the avalanches are sequentially distributed, in turn, in the urban development they may happen contemporaneously. These differences are considered in-depth in the following section where the model is explained.

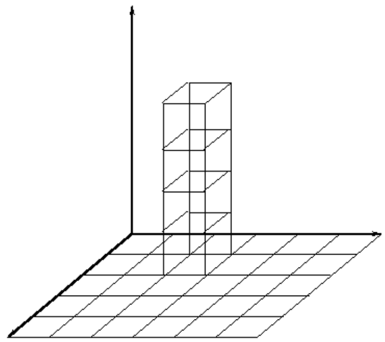

Figure 2: The spatial patter in 3-D. The distances are calculated, as row flies, in 2-D. The building in the third dimension is submitted to the constraint that the underlying cell was already built.

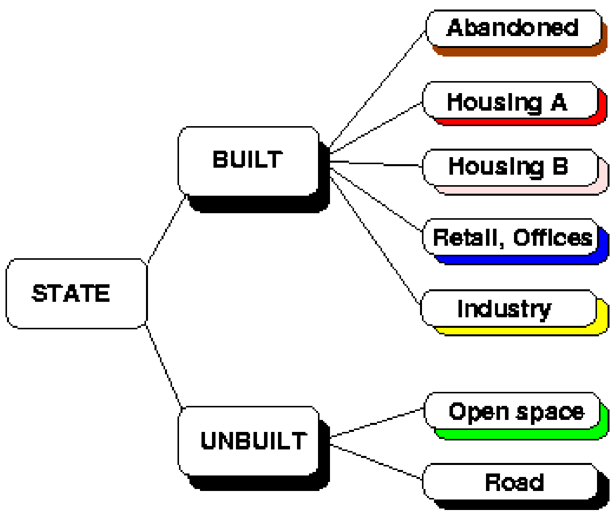

Figure 3: The allowed states for each cell. States are divided in build and un-built. The built states include two types of housing, A, and B, retail and offices, and industry. The income of housing A is considered higher then the income of housing B. In addition a cell is abandoned when it is built but empty as use. The unbuilt states include roads and open spaces.

The model

The model is organized in a 3-d squared grid of cubic cells, as in [3] (figure 2). Each cell can take a state, or can be occupied by an use (figure 3) and the model dynamic is based on the transition of each cell from one state, or use, to another. The global dynamic is constrained to total values for each use established exogenously [4]. If the current number of cell for a specified use is lower than the established, then some cells are stimulated to change state.

While the global values are constrained, the spatial distribution of it is totally managed by the model. In essence the functioning of the model is the following. The grains are

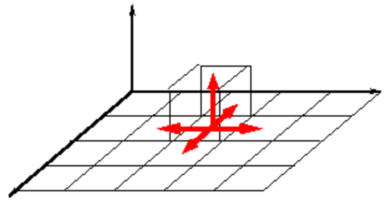

Figure 4: The distribution of the grains into the four surrounding cells and in the upper cell.

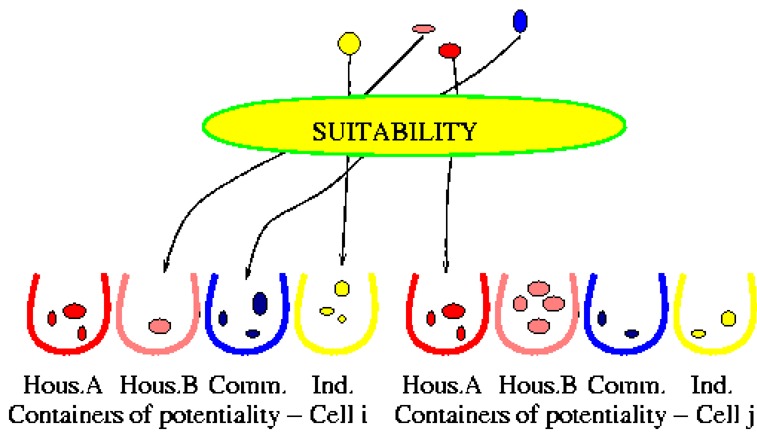

Figure 5: The grains fall down in the containers of cell i or j depending of the suitability of each cell for the specified use.

specialized in relation to the relevant uses, i.e.: housing A and B, retail, and industry. In other words each cell has a number of containers of grains equal to the number of possible relevant uses (figure 5). The grains in each container represent the potentiality for a cell to be utilized for the corresponding use. At each step a grain is added to some cell in dependence to the difference: global desired quantity, existent quantity of each use. These grains are distributed in relation to the suitability of this cell for the use in question. When the number of grains in a container of a cell related to a use reaches the threshold, set equal to 5, the cell is assigned to the use and the potentiality of all the container of the cell is decreased by a quantity equal to the threshold. In fact, after the change of state, the potentiality of further variation in the cell is null or negative. The potentiality related to the assigned use is distributed into the surrounding cells. These surrounding cells are the four bordering plus the upper cell, because the pattern is in 3-D (see figure 4). In addition the threshold is equal to the number of cells in which the grains are distributed.

Let us consider the method for the distribution of grains. In the sandpile model the grains are assigned randomly, while in the present model they are assigned in relation to the suitability of a cell for the specified use (figure 5). The suitability is calculated in relation to the surrounding uses in a radius of 6 cells, and depending on the distance from the central cell. The slope and the nearby to the roads are considered ([4]), as well as the cost of building in relation to the floor [3]. In addition, because the assignment of a grain is based on the comparison of the suitability for different uses, a weight accounting for the importance of use, or in other words for the ability of the use to compete in the land market, is included. Finally the result is multiplied by a random factor which simulates the imperfect knowledge. In conclusion the suitability for a cell c_{ijk} to be in state p is calculated by using the following equation:

$$S_p = \frac{\sum_{q,d} m_{p,q,d} I_d - s_{ij}}{C_k} W_p r \qquad (1)$$

where:

S_p is the suitability for the state p in cell c_{ijk};

$m_{p,q,d}$ is the weight connected to the cells in state q at distance d from c_{ijk} in relation to state p;

$I_d = 1$ if he state of the cell distant d from c_{ijk} is equal to q, $I_d = 0$ otherwise;

C_k is the building cost for a cell at floor k;

s_{ij} represents the difficulty to build in relation to the slope of ground;

W_p is the weight related to the state p;

r is a random factor: $r = 1 + [-ln(rand)]^\alpha$, $\alpha = 3$.

In order to avoid an huge computation time, the suitability is calculated for a set of cells in which are included abandoned cells plus some cells chosen at random among unbuilt cells as well as built and assigned to an use from an established period which is about 200 steps. This set, which in the following experiments, represents about 10% of the total quantity of cells, does not include cells in critical state i.e. cells having a potentiality equal to the threshold. In other words the addition of grains does not disturb the acting avalanches. After having calculated the suitability, the grains are distributed with the following method. Grains are assigned beginning by the maximum suitability till the global quantity of each grains, exogenously established, in relation to the desired quantity of each use, is reached. Finally cells are abandoned after an established period, and an abandoned cells is demolished if it is not occupied after an established period. The entire process is represented in figure 6.

Results

The experiments have been performed on a squared grid of 50X50x10 cells of 200 meters sized. In order to limit the number of grains in relation to the stimulated avalanches, each period, which is supposed to correspond to one year, is divided in 10 sub-periods. In each experiment, iterations have been 4000, corresponding to 400 years. In the first 200 years (i.e. 2000 steps) the number of built cells grows, while in the second part the number of built cells is stable, and only the abandoned cells are replaced by the model dynamic.

The maximum quantity of each use, as well as the values of the weights W_p are established as in table 1. The quantity of each use is supposed equal to 1 at the beginning and

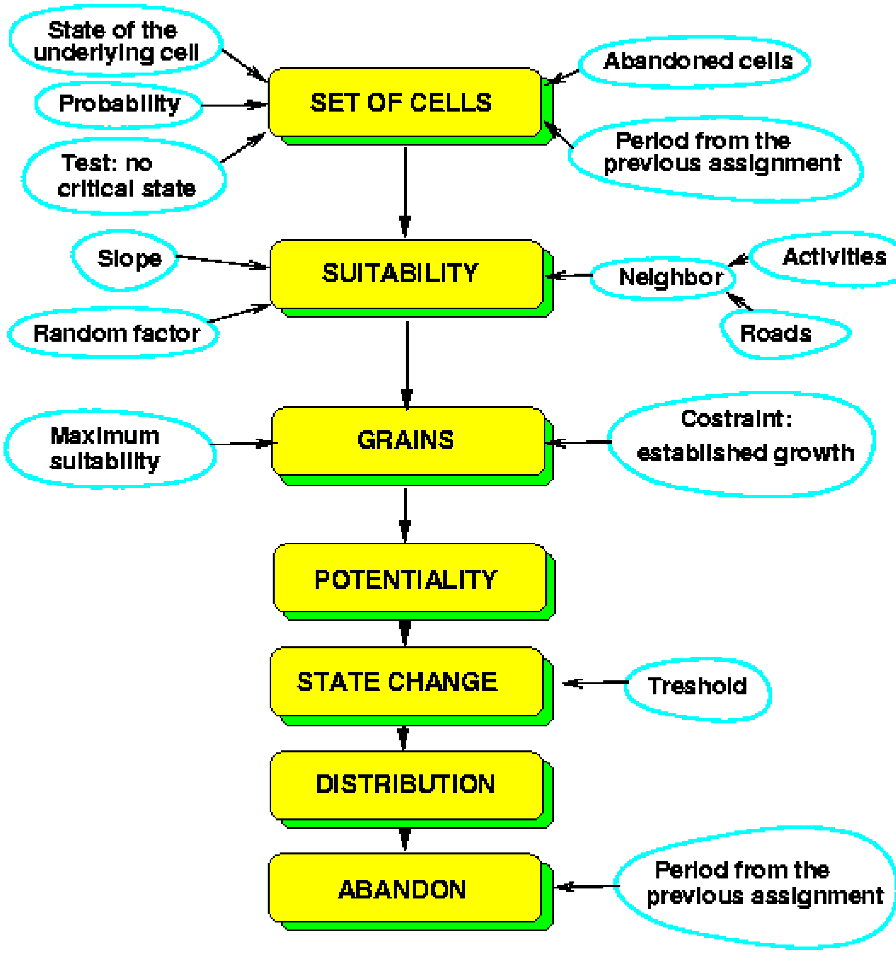

Figure 6: The process for the change of the state of a cell.

Table 1: The quantity of cells per use, and the corresponding quantities of inhabitants and employees. In the last column are included the values utilized for the weights W_p

Use	Cells	Inhabitants or employees per cell	Total inhabitants or employees	W_p
Housing A	200	500	100 000	10
Housing B	200	500	100 000	5
Retail, offices	60	600	36 000	50
Industry	10	300	42 000	1

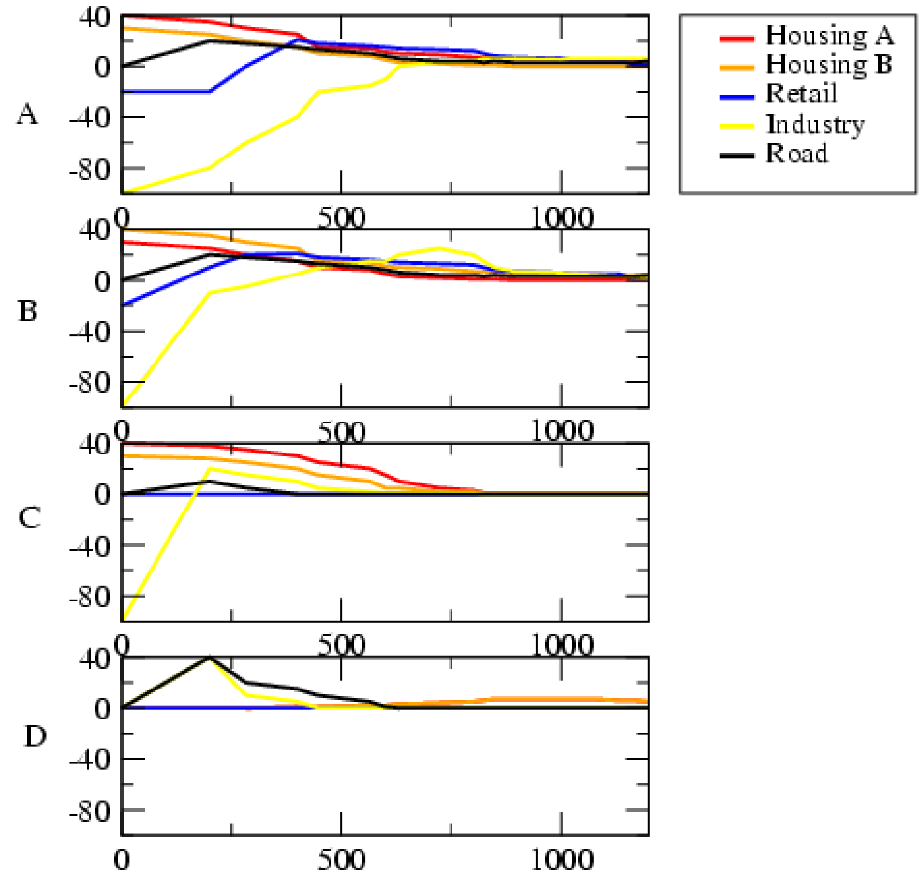

Figure 7: Variation of weights $m_{p,q,d}$. Graph A: X axis, distance (d), Y axis weight of cell in state q (states are listed in the legend) in connection with Housing A use. Graph B: Y axis weight of cell in state q in connection with Housing B use. Graph C: Y axis weight of cell in state q in connection with retail use. Graph D: Y axis weight of cell in state q in connection with industrial use.

increases linearly till the maximum after 200 years. In fact a seed is established almost in the center of the area. At each period the expected quantity of each use is calculated. This quantity is utilized in order to establish the number of grain to distribute. The values of the parameters $m_{p,q,d}$ are shown in figure 7, and the resulting spatial pattern is shown in figure 8.

Because the dynamic is based on the transfer of grain of potentiality from one cell to the surrounding cells the size of each avalanche is calculated by recording the chain of causation from the start up cell to the other cells. In order to evaluate the distribution of the size of avalanches, these have been ranked by size. In other words we have estimated the cumulative distribution function (CDF). The probability distribution function (PDF) can be obtained by increasing the exponent of the CDF by one. The size has been plotted in relation to the rank, and the result is shown in figure 9. The estimated function is $s \propto r^{-0.28}$, where s is the size of the avalanche, calculated by using the number of cells included in the avalanche, and r the rank. The CDF is $P(s' > s) \propto s^{-1/0.28}$ and the PDF

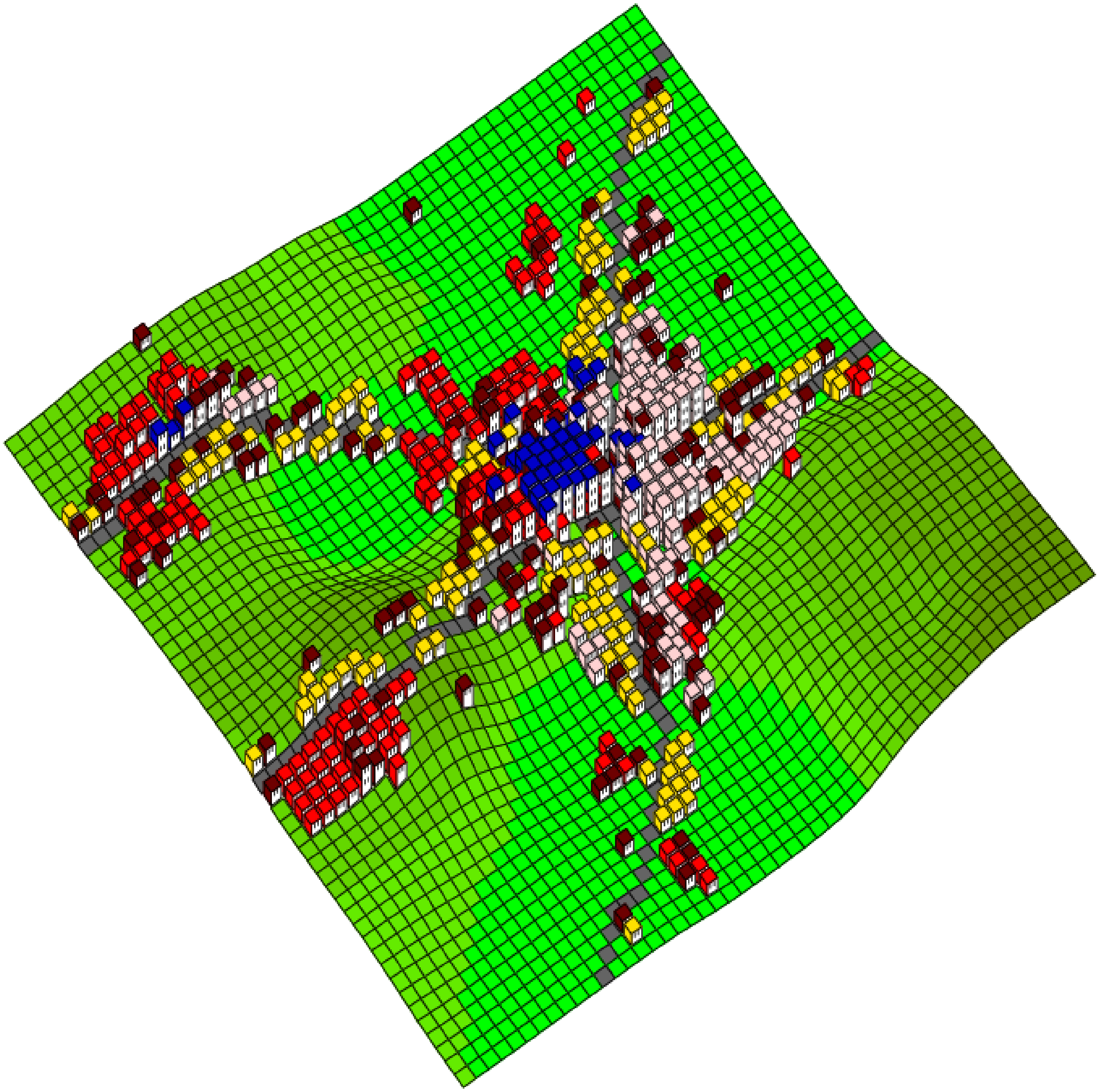

Figure 8: The spatial pattern after 400 years. Red: housing A, pink: housing B, blue: retail, yellow: industry, brown: abandoned.

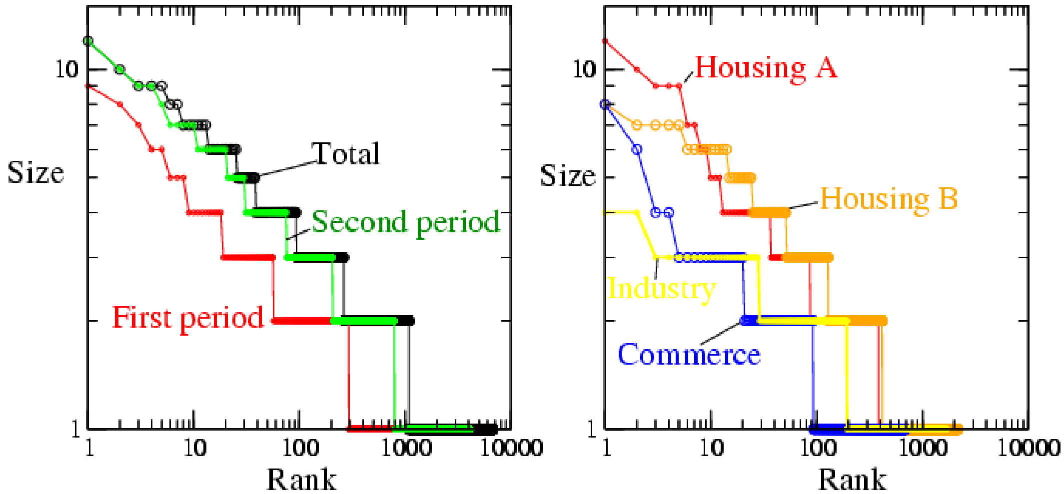

Figure 9: The rank-size distribution of avalanches. X axis: rank, Y axis: size. Left side, black: the rank-size distribution obtained considering all the avalanches, green: the rank size distribution obtained considering the avalanches happened during the growth period, red: the rank size distribution obtained considering the avalanches happened during the stability period. Right side, the rank-size distribution of the avalanches per use, red: housing A, green: housing B, blue: retail, yellow: industry.

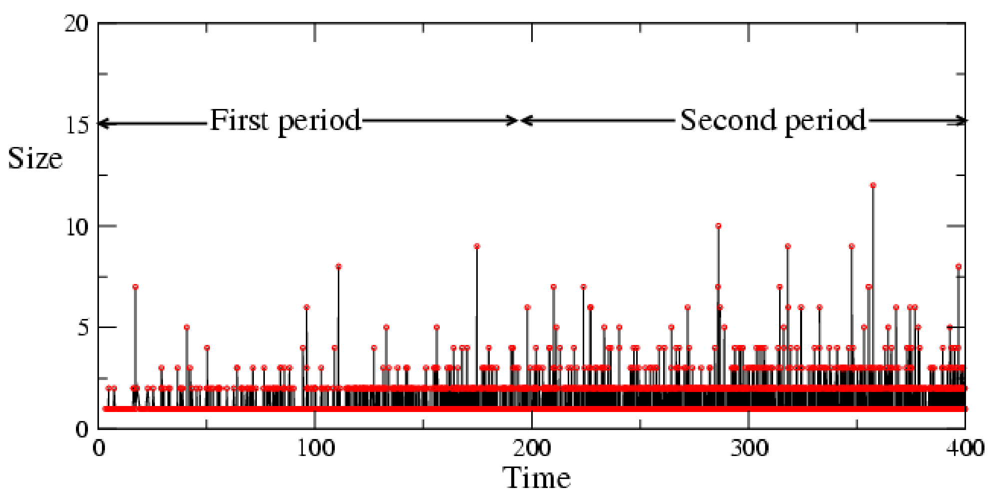

Figure 10: The temporal series of avalanches. X axis: time, Y axis: size of avalanches.

is $P(s) \propto s^{-(1/0.28+1)}$. This result means that avalanches of great size are very limited in relation to the small avalanches. In addition from figure 10 it results that during the period of stability the size of avalanches increases.

Discussion

In order to evaluate the impact of the sandpile method, a probability to distribute the grains of potentiality in the 5 surrounding cells, has been included, as parameter, in the model. The application of this parameter results in a change of the urban cluster. This change is evaluated by using the centroid of the urban cluster during the simulation. Twenty simulations have been performed by varying the seed of the random number generator. The first ten by using a probability to distribute the grains equal to one, i.e. by using the normal model, and the second ten by using a probability equal to 0.1. The resulting spatial pattern of one of the second set of experiments is shown in figure 11, while in figure 12 are shown the paths of the centroids in the first and second set of experiments during the steps of the simulations. As figure 12 shows, the paths of the second set of experiments are less scattered. In essence the more the process of distribution of grains is activated the more the final result is dependent on initial conditions. In other words the chaotic behavior of the dynamic system depends strictly of the chains of causation established by the sandpile method.

Conclusion

The sandpile method has been applied to the simulation of the urban development. The urban dynamic can be well simulated as a system in which the steady state is characterized by a self-organized criticality, and it has been shown that the chaotic behavior of the simulated urban dynamic depends of the chain of causations generated by the sandpile model.

Acknowledgments

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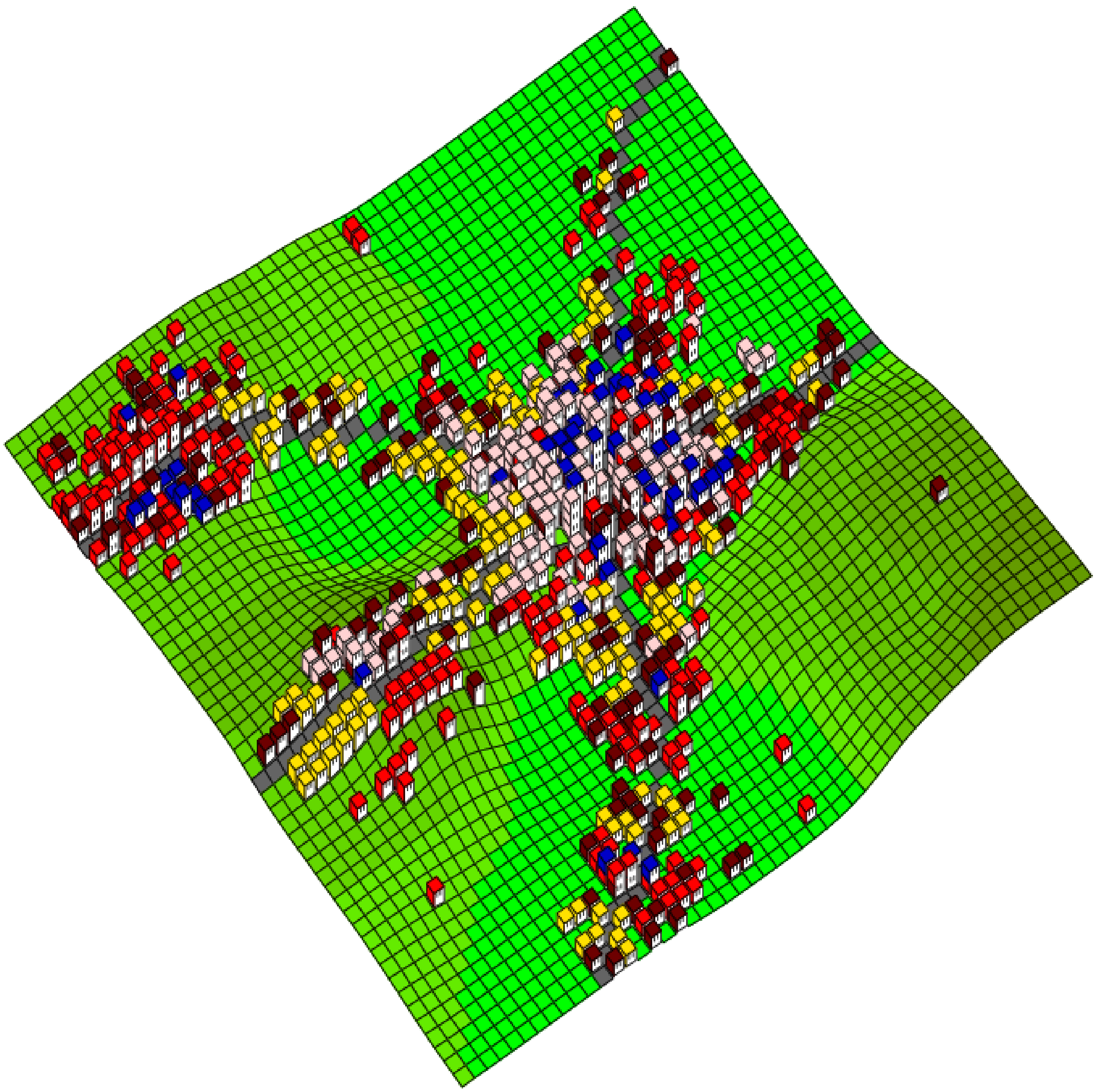

Figure 11: The spatial pattern after 400 years. Red: housing A, pink: housing B, blue: retail, yellow: industry, brown: abandoned.

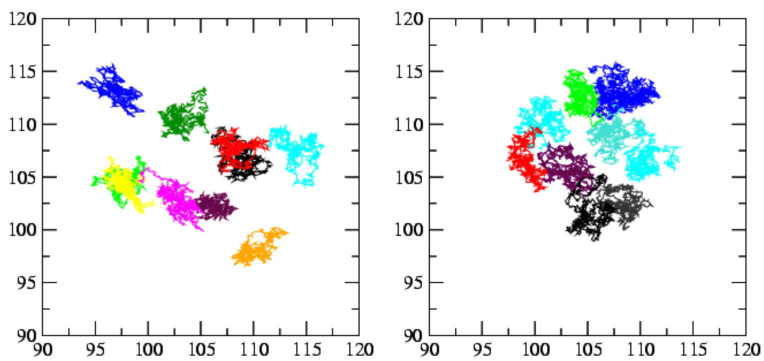

Figure 12: The path of the centroid of the urban cluster, in the final period of the simulation. Ten simulations obtained by varying the seed of the random number generator. Left side probability equal 1, high variability. Right side probability equal 0.1, low variability.